G.C.E. (A.L.) Examination - 2019
10 - Combined Mathematics II (Part A)
(Old Syllabus)

Distribution of Marks

Paper II

Part A : 10 × 25 = 250

Part B : 05 × 150 = 750

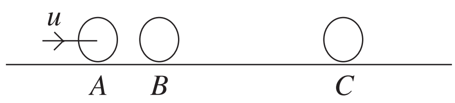
Total = 1000 / 10

Paper II Final Mark = 100

- **Part B is common to both Old Syllabus and New Syllabus.**

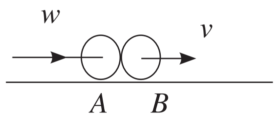
1. Three particles A, B and C, each of mass m, are placed in that order, in a straight line on a smooth horizontal table. The particle A is given a velocity u such that it collides directly with the particle B. After colliding with the particle A, the particle B moves and collides directly with the particle C. The coefficient of restitution between A and B is e. Find the velocity of B after the first collision.

The coefficient of restitution between B and C is also e. Write down the velocity of C after its collision with B.



Applying $\underline{I} = \Delta(m\underline{v})$,

for A and B (1^{st} collision) $\rightarrow$:



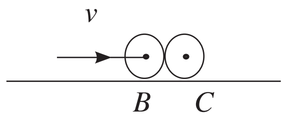
$$0 = mv + mw - mu$$ (5)

$\Rightarrow v + w = u$ ———— (i)

Newton's law of restitution :

$v - w = eu$ ———— (ii) (5)

$\therefore$ (i) + (ii) $\Rightarrow v = \dfrac{(1+e)}{2} u$ (5)



$\therefore$ velocity of B after 1^{st} collision $= \dfrac{1}{2}(1+e)\,u$.

Replacing u by v, we get the velocity of C after its collision with B $= \dfrac{1}{2}(1+e)\,v$ (5)

$= \dfrac{1}{4}(1+e)^2\,u$ (5)

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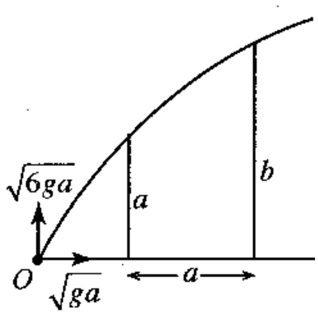
2. A particle is projected from a point O on a horizontal floor with a velocity whose horizontal and vertical components are $\sqrt{ga}$ and $\sqrt{6ga}$, respectively. The particle just clears two vertical walls of heights a and b which are at a horizontal distance a apart, as shown in the figure. Show that the vertical component of the velocity of the particle when it passes the wall of height a is $2\sqrt{ga}$. Show further that $b = \frac{5a}{2}$.

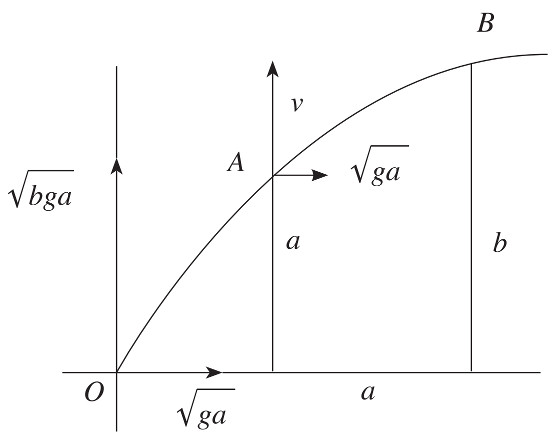


Suppose that the particle passes the wall of height a with vertical velocity component v.

From O to A, $\uparrow$ $v^2 = u^2 + 2as$:

$v^2 = 6ga - 2g \cdot a = 4ga$ (5)

$\therefore v = 2\sqrt{ga}$ (5)



If it passes the second wall, after a further time T, then by applying

$s = ut + \frac{1}{2}at^2$ from A to B, $\rightarrow$ and $\uparrow$, we get

$a = \sqrt{ga} \cdot T$, (5)

and $b - a = 2\sqrt{ga} \cdot T - \frac{1}{2}gT^2$ (5)

Eliminating T : $b - a = 2\sqrt{ga} \cdot \sqrt{\frac{a}{g}} - \frac{1}{2}g \cdot \frac{a}{g}$

$\therefore b = a + 2a - \frac{a}{2}$

i.e. $b = \frac{5a}{2}$ (5)

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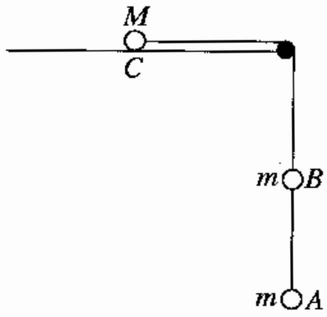
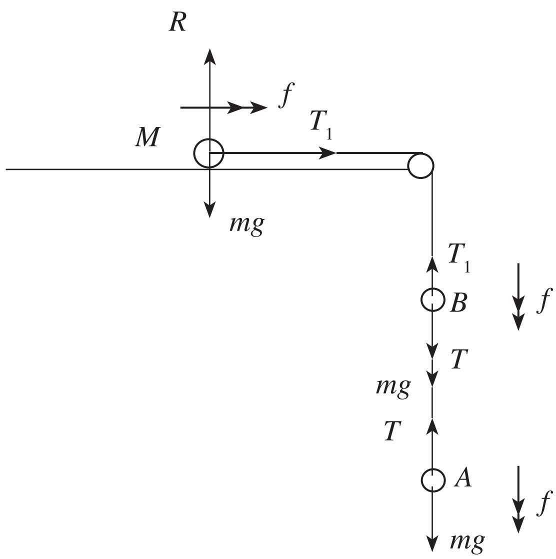
3. In the figure, A, B and C are particles of masses m, m and M, respectively. The particles A and B are connected by a light inextensible string. The particle C, lying on a smooth horizontal table, is connected to B by another light inextensible string passing over a smooth small pulley fixed at the edge of the table. The particles and the strings all lie in the same vertical plane. The system is released from rest with the strings taut. Write down equations sufficient to determine the tension of the string joining A and B.





Applying $\underline{F} = m\underline{a}$

For A, $\downarrow$ $mg - T = mf$ (5)

For B, $\downarrow$ $T + mg - T_1 = mf$, (5)

For C, $\rightarrow$ $T_1 = Mf$ (5)

Forces (5)

Accelerations (5)

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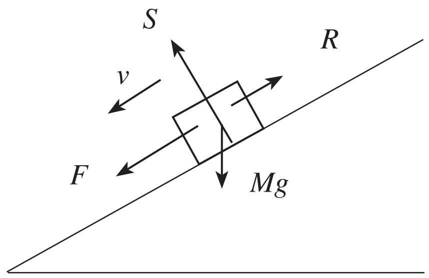
4. A car of mass M kg and constant power P kW moves downwards along a straight road of inclination α to the horizontal. There is a constant resistance of $R\,(> Mg\sin\alpha)$ N to its motion. At a certain instant, the acceleration of the car is a m s^{-2}. Find the velocity of the car at this instant.
Deduce that the constant speed with which the car can move downwards along the road is $\dfrac{1000P}{R - Mg\sin\alpha}$ m s^{-1}.



When the speed of the car is v ms^{-1}

tractive force $F = \dfrac{1000\,P}{v}$ (5)

At the instant when the acceleration is a ms^{-2},

Applying $\underline{F} = m\underline{a}$:

↙ $F + Mg\sin\alpha - R = Ma.$ (10)

$$\Rightarrow \frac{1000\,P}{v} + Mg\sin\alpha - R = Ma$$

$$\therefore \quad v = \frac{1000\,P}{R - Mg\sin\alpha + Ma}$$ (5)

When the car is moving with constant speed,

$a = 0$ and the value of constant speed

$$v = \frac{1000\,P}{R - Mg\sin\alpha}.$$ (5)

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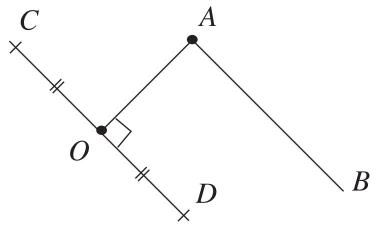
5. In the usual notation, let $2\mathbf{i}+\mathbf{j}$ and $3\mathbf{i}-\mathbf{j}$ be the position vectors of two points A and B, respectively, with respect to a fixed origin O. Find the position vectors of the two distinct points C and D such that $A\hat{O}C = A\hat{O}D = \frac{\pi}{2}$ and $OC = OD = \frac{1}{3}AB$.



Note that

$\overrightarrow{OA} = 2\mathbf{i} + \mathbf{j}$

$\overrightarrow{OB} = 3\mathbf{i} - \mathbf{j}$

$\therefore \overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -(2\mathbf{i} + \mathbf{j}) + (3\mathbf{i} - \mathbf{j})$

$= \mathbf{i} - 2\mathbf{j}$ (5)

$\therefore AB = \sqrt{1+4} = \sqrt{5}$

Let $\overrightarrow{OC} = xi + yi$

Since $\overrightarrow{OA} \perp \overrightarrow{OC}$, $(2i + j) + (x\mathbf{i} + y\mathbf{j}) = 0$

$\therefore y = -2x$ (5)

Since $OC = \frac{1}{3}AB$, $\sqrt{x^2 + 4x^2} = \frac{1}{3}\sqrt{5}$ (5)

$\therefore x^2 = \frac{1}{9}$.

These equations are valid for the coordinates of D as well.

So, $x = \pm\frac{1}{3}$

$\Rightarrow$ $x = \frac{1}{3}$, $y = -\frac{2}{3}$ (5) $\quad$ $x = -\frac{1}{3}$, $y = \frac{2}{3}$ (5)

Hence the vectors C and D are $\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j}$ and $-\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j}$.

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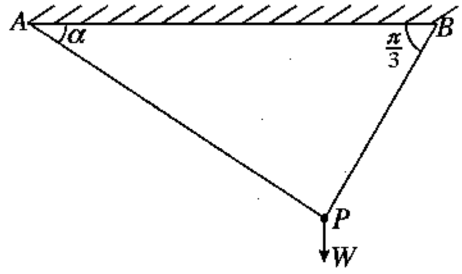
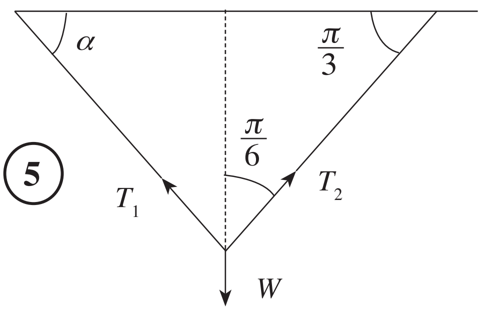
6. A particle P of weight W, suspended from a horizontal ceiling by two light inextensible strings AP and BP making angles α and $\frac{\pi}{3}$ with the horizontal, respectively, is in equilibrium as shown in the figure. Find the tension in the string AP in terms of W and α.

Hence, find the minimum value of this tension and the corresponding value of α.



By Lami's theorem

$$\frac{T_1}{\sin\frac{\pi}{6}} = \frac{W}{\sin\left(\frac{\pi}{2} - \alpha + \frac{\pi}{6}\right)} \quad (10)$$



(5)

$$\therefore T_1 = \frac{W}{2\sin\left(\frac{\pi}{3} + \alpha\right)} \quad (5)$$

Hence the minimum value of the tension T_1 in $AP = \frac{W}{2}$, and

the value of α corresponding to minimum of T_1 is, $\alpha = \frac{\pi}{6}$. (5)

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7. Let A and B be two events of a sample space Ω. In the usual notation, it is given that $P(A)=\frac{3}{5}$, $P(A\cap B)=\frac{2}{5}$ and $P(A'\cap B)=\frac{1}{10}$. Find $P(B)$ and $P(A'\cap B')$; where A' and B' denote complementary events of A and B, respectively.

$$P(B) = P\big((A\cap B)\cup(A'\cap B)\big) = P(A\cap B)+P(A'\cap B) \quad (5)$$

$$= \frac{2}{5}+\frac{1}{10}.$$

$$\therefore P(B) = \frac{1}{2}. \quad (5)$$

$$P(A'\cap B') = P\big((A\cup B)'\big)$$

$$= 1 - P(A\cup B) \quad (5)$$

$$= 1 - [\,P(A)+P(B)-P(A\cap B)\,] \quad (5)$$

$$= 1 - [\frac{3}{5}+\frac{1}{2}-\frac{2}{5}]$$

$$= 1 - \frac{7}{10}$$

$$\therefore P(A'\cap B') = \frac{3}{10} \quad (5)$$

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8. A bag contains 3 red balls and 6 black balls which are identical in all aspects except for their colour. Two balls are drawn at random from the bag, one at a time, without replacement. Find the probability that the second ball drawn is black.

Find also the probability that the first ball drawn is red, given that the second ball drawn is black.

P (Second ball drawn is black)

$= P(1^{st}\text{ red and }2^{nd}\text{ Black}) + P(1^{st}\text{ black and }2^{nd}\text{ Black})$ (5)

$= \frac{3}{9} \times \frac{6}{8} + \frac{6}{9} \times \frac{5}{8}$ (5)

$= \frac{2}{3}$ (5)

$P(1^{st}\text{ red} \mid 2^{nd}\text{ black})$

$= \dfrac{P(1^{st}\text{ red and }2^{nd}\text{ Black})}{P(2^{nd}\text{ Black})}$ (5)

$= \dfrac{\frac{3}{9} \times \frac{6}{8}}{\frac{2}{3}}$

$= \frac{3}{8}$ (5)

9. **Five positive integers each of which is less than 5, have two modes, one of which is 3. Their mean, and median are both equal to 3. Find these five integers.**

With median = 3, and two distinct modes, five numbers which are less five, in ascending order can be arranged in the following two possible ways.

$a, a, 3, 3, 4$ (5)

$b, 3, 3, 4, 4$ (5)

Since their sum is 15 as the mean is 3,

we have, $2a + 10 = 15$; $a = \frac{5}{2}$, # (5)

or $b + 14 = 15$; $b = 1$. (5)

$\therefore$ Five numbers are 1, 3, 3, 4, 4 (5)

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10. A frequency distribution is given in the following table:

Range of values	0 – 5	5 – 10	10 – 15	15 – 20
Frequency	8	10	7	5

Find the mode of this distribution.

The mode of the distribution of values obtained by multiplying each value of the above distribution by a constant k and then adding 7 to it is 21. Find the value of k.

$$M = L_M + C\left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)$$

$$= 5 + 5\left(\frac{2}{2+3}\right) \quad (10)$$

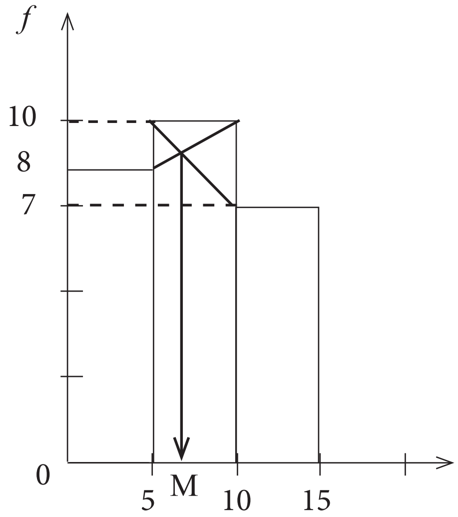
$$= 7 \quad (5)$$

New mode is 21.

$\therefore 21 = k(7) + 7 \quad (5)$

$\therefore k = 2 \quad (5)$

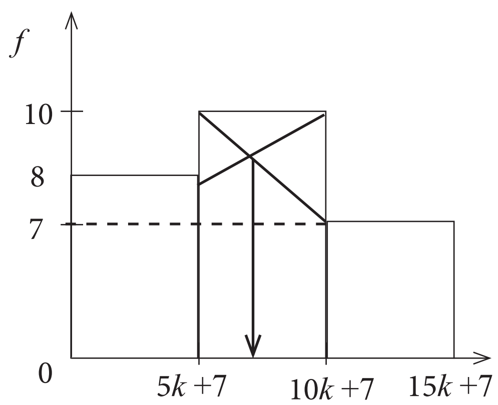
Aliter



M - Mode

$\left(\frac{M-5}{10-8}\right) = \left(\frac{10-M}{10-7}\right)$ (10) $\Rightarrow M = 7$ (5)

Aliter



$$\frac{21-(5k+7)}{10-8} = \frac{(10k+7)-21}{10-7} \quad (5)$$

$\Rightarrow k = 2 \quad (5)$

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